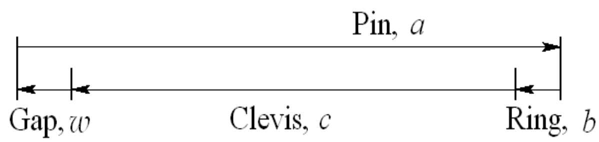
Homework 1 Solutions

Problem 1



$w_{\max} = 0.05$ in, $w_{\min} = 0.004$ in

$$\bar{w} = \frac{0.05 + 0.004}{2} = 0.027 \text{ in}$$

Thus, $\Delta w = 0.05 - 0.027 = 0.023$ in, and then, $w = 0.027 \pm 0.023$ in.

$$\bar{w} = \bar{a} - \bar{b} - \bar{c}$$

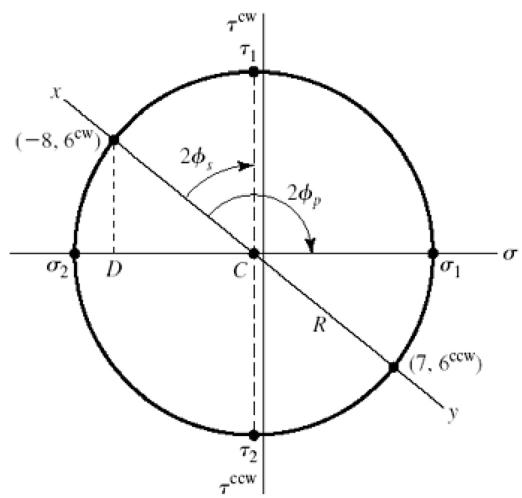
$$0.027 = \bar{a} - 0.042 - 1.5$$

$$\bar{a} = 1.569 \text{ in}$$

$$t_w = \sum t_{\text{all}} \quad \Rightarrow \quad 0.023 = t_a + 0.002 + 0.005 \quad \Rightarrow \quad t_a = 0.016 \text{ in}$$

Thus, $a = 1.569 \pm 0.016$ in *Ans.*

Problem 2



$$C = \frac{-8+7}{2} = -0.5 \text{ MPa}$$

$$CD = \frac{8+7}{2} = 7.5 \text{ MPa}$$

$$R = \sqrt{7.5^2 + 6^2} = 9.60 \text{ MPa}$$

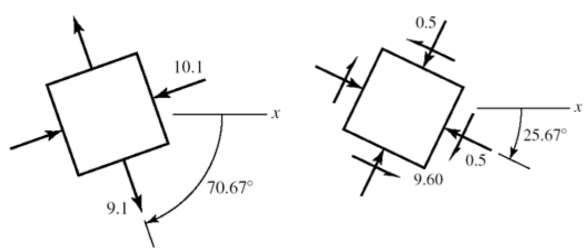
$$\sigma_1 = 9.60 - 0.5 = 9.10 \text{ MPa}$$

$$\sigma_2 = -0.5 - 9.6 = -10.1 \text{ Mpa}$$

$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{7.5}{6}\right)\right] = 70.67^\circ \text{ cw}$$

$$\tau_1 = R = 9.60 \text{ MPa}$$

$$\phi_s = 70.67^\circ - 45^\circ = 25.67^\circ \text{ cw}$$



Problem 3

(a)

$$T_2 = 0.15T_1$$

$$\sum T = 0 = (1800 - 270)(200) + (T_2 - T_1)(125) = 306\left(10^3\right) + 125\left(0.15T_1 - T_1\right)$$

$$306\left(10^3\right) - 106.25T_1 = 0 \quad \Rightarrow \quad T_1 = 2880 \text{ N} \quad \textit{Ans.}$$

$$T_2 = 0.15(2880) = 432 \text{ N} \quad \textit{Ans.}$$

(b)

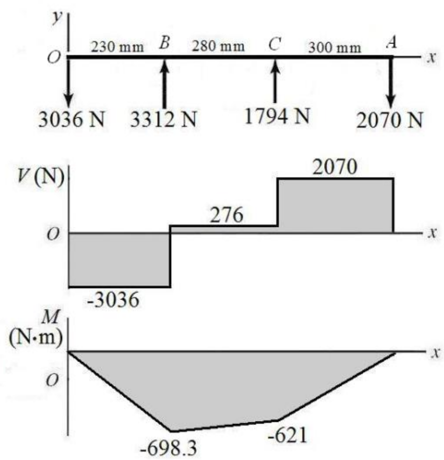
$$\sum M_O = 0 = 3312(230) + R_C(510) - 2070(810)$$

$R_C = 1794\ \text{N}$ *Ans.*

$$\sum F_y = 0 = R_O + 3312 + 1794 - 2070$$

$R_O = -3036\ \text{N}$ *Ans.*

(c)



(d) The maximum bending moment is at x = 230 mm, and is M = –698.3 N·m. Since the shaft rotates, each stress element will experience both positive and negative bending stress as it moves from tension to compression. The torque transmitted through the shaft from A to B is T = (1800 – 270)(0.200) = 306 N·m. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(698.3)}{\pi(0.030)^3} = 263\left(10^3\right)\ \text{Pa} = 263\ \text{MPa} \quad \textit{Ans.}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(306)}{\pi(0.030)^3} = 57.7\left(10^6\right)\ \text{Pa} = 57.7\text{MPa} \qquad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{263}{2} \pm \sqrt{\left(\frac{263}{2}\right)^2 + \left(57.7\right)^2}$$

$\sigma_1 = 275\ \text{MPa}$ *Ans.*

$\sigma_2 = -12.1\ \text{MPa}$ *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{263}{2}\right)^2 + \left(57.7\right)^2} = 144\ \text{MPa} \qquad \textit{Ans.}$$

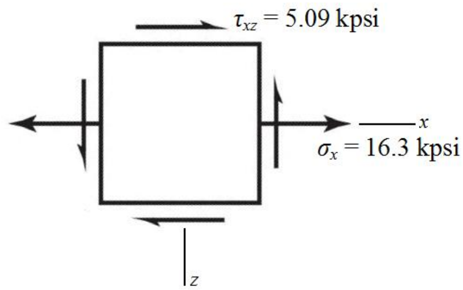
Problem 4

(a) Rod AB experiences constant torsion throughout its length, and maximum bending moment at the wall. Both torsional shear stress and bending stress will be a maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at the wall, at either the top (compression) or the bottom (tension) on the *y* axis. We will select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$$\sigma_x = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(8)(200)}{\pi(1)^3} = 16\,297\text{ psi} = 16.3\text{ kpsi}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(5)(200)}{\pi(1)^3} = 5093\text{ psi} = 5.09\text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \frac{16.3}{2} \pm \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2}$$

$\sigma_1 = 17.8$ kpsi *Ans.*

$\sigma_2 = -1.46$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2} = 9.61\text{ kpsi} \qquad \textit{Ans.}$$